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Stochastic particle annihilation: a model of state reduction in relativistic quantum field theory

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Abstract

A model of state reduction in relativistic quantum field theory involving a nonlinear stochastic extension of Schrödinger's equation is outlined. The eigenstates of the annihilation operator are chosen as the preferred basis onto which reduction occurs. These are the coherent states which saturate the bound of the Heisenberg uncertainty relation, exhibiting classical-like behaviour. The quantum harmonic oscillator is studied in detail before generalizing to relativistic scalar quantum field theory. The infinite rates of increase in energy density which have plagued recent relativistic proposals of dynamical state reduction are absent in this model. This is because the state evolution equation does not drive particle creation from the vacuum. The model requires the specification of a preferred sequence of space-like hyper-surfaces supporting the time-like state evolution. However, it is shown that the choice of preferred surfaces has no effect on perturbative results to second order in the coupling parameter. It is demonstrated how state reduction to a charge density basis can be induced in fermionic matter via an appropriate coupling to a bosonic field undergoing this mechanism.

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1. Introduction

Much of the peculiar behaviour associated with quantum physics results from the fact that, although a quantum system can be in a superposition of different states, whenever we make measurements involving macroscopic apparatus, a definite state is always registered. The transition from a superposition to a definite state is not described by Schrödinger's equation. How then, if the constituents of the apparatus are also described by Schrödinger's equation, does this quantum state reduction come about?

Stochastic generalizations of Schrödinger's equation have been proposed by a number of authors in answer to the problem of measurement [1–5] (for a review see [6, 7]). The key idea is that measurement is understood as the realization of a random process in the Hilbert

space of state vectors where unwanted superpositions of states are unstable. The appeal of these models rests on two fundamental properties: (i) they reproduce quantum effects on small scales with negligible modification to standard quantum theory, and (ii) they lead to rapid, objective state vector collapse on large scales with probabilities given by the laws of standard quantum mechanics. The result is that superpositions of states for macroscopic objects are suppressed whilst individual particles continue to behave according to quantum theory.

The usual approach is to substitute Schrödinger's equation with a quantum state diffusion equation of the form

$$\mathrm{d}|\phi_t\rangle = (C\,\mathrm{d}t + \mathbf{A}\cdot\mathrm{d}\mathbf{X}_t)\,|\phi_t\rangle. \tag{1}$$

Here $\{\mathbf{X}_t\}$ is a (vector-valued) Itô process and $\mathbf{A}$, C are operators (the Schrödinger equation can be recovered by setting $C = -\mathrm{i}H$ and $\mathbf{A} = 0$). With appropriate choices for the drift and volatility of $\{\mathbf{X}_t\}$ the quantum state typically evolves into an eigenstate of the operator $\mathbf{A}$. The choice of $\mathbf{A}$ leads to a preferred basis. In the quantum-mechanical case, the standard choice is a locally averaged position state basis in order to reproduce the definite localization of objects at the classical scale. Another idea is to use an energy state basis [8–10]. These models have the desirable property that energy is conserved in expectation. A general solution to the energy-based state diffusion with time-dependent coupling has recently been found [11].

At present, non-relativistic proposals are seen to have sufficiently negligible effects on the quantum scale in order to be indistinguishable from standard quantum theory for current experimental technologies [12]. At the same time these proposals offer a consistent understanding of classical and quantum domains. However, so far, relativistic field theoretic formulations generally predict an infinite rate of particle creation due to the coupling of a classical stochastic field to a quantum scalar field [6, 13–15]. Some previous attempts to resolve this problem have involved modifying the stochastic field to prevent high-energy excitations [7, 16], or coupling the noise source not locally to the quantum field but to the integral of quantum fields over some spacetime region [17]. A quantum-mechanical model for a relativistic particle has been developed in [18] although this model does not include interactions.

In this paper, we outline an alternative proposal in which the stochastic field is coupled only to the annihilation operators of the quantum scalar field (via a local interaction term). The scalar field cannot then be excited by the stochastic field. As a consequence, the infinite rates of energy increase are avoided. Instead we see an expected energy loss to the stochastic field which can be controlled to a negligibly small level by an appropriate choice for the coupling parameter. A related idea has been employed in [19] to control energy increase in models of non-relativistic state reduction.

We find that in order to construct a satisfactory model of state reduction in relativistic quantum field theory, we must assume a preferred sequence of space-like hyper-surfaces supporting the evolution of the quantum state. The reason is that the stochastic field is coupled to local operators which do not commute at space-like separation. The state evolution equation is therefore path dependent. The fixed sequence of space-like hyper-surfaces constrains the evolution such that only one path is possible, ensuring a well-defined evolving state. We do not propose a rule for how the surfaces are chosen and regard them as a hidden property of the state.

Our state evolution equations are of relativistically invariant form so that all observers will agree on outcomes. However, the choice of surface is responsible for identifying a preferentially selected local frame. The idea that dynamical reduction models might break Lorentz invariance in this way has been suggested before by Pearle [20], who considered a stochastic field coupled to a generalized mass–density field which does not commute at

space-like separation. There it was shown that the commutator decays on a length scale corresponding to the particle's Compton wavelength, providing a sense in which the model is quasi-relativistic.

By performing perturbative calculations involving an expansion in the coupling parameter, we are able to quantify the effect of a particular choice of the space-like hyper-surfaces. We find that the choice has no effect on the lowest order expressions describing state reduction. This offers an alternative way to understand the quasi-relativistic nature of this type of model.

We will see that the quantum state evolves towards the eigenstates of the annihilation operators. In quantum mechanics these are well understood as coherent states (see e.g. [21]). The coherent states have long been regarded as a close quantum approximation to idealized classical states and therefore constitute a natural choice for the preferred basis states in a quantum state reduction model.

The paper is organized as follows. In section 2, we demonstrate the state reduction mechanism for the simple case of a quantum harmonic oscillator. By analysing the quantum variance processes we are able to demonstrate that state reduction occurs, and to estimate the associated reduction timescale. We also examine how the expectation of energy evolves and demonstrate that initial quantum probabilities match with the probabilities of stochastic outcomes in a simple example. We conclude the section with some numerical results which confirm our analysis.

In section 3, we extend the formalism to a relativistic quantum scalar field. We adopt the interaction picture of Tomonaga and Schwinger [22, 23] to describe a state defined on some space-like hyper-surface evolving in a time-like manner. Once we have examined this picture in detail, we proceed to demonstrate the reductive properties. We show how this mechanism of state reduction for a bosonic field could induce a state reduction to some charge state basis in a fermionic field. We end in section 4 with some concluding remarks.

2. Quantum-mechanical harmonic oscillator

The device we shall use to represent quantum state reduction will be presented for the case of $(0+1)$-dimensional scalar field theory, i.e. the quantum-mechanical harmonic oscillator. The commutation relation between position and momentum is given by $[x, p] = \mathrm{i}$. We define creation and annihilation operators in the standard way as follows:

$$\begin{cases} a = \sqrt{\dfrac{\omega}{2}}(x + \mathrm{i}p\omega^{-1}) \\ a^\dagger = \sqrt{\dfrac{\omega}{2}}(x - \mathrm{i}p\omega^{-1}) \end{cases} \Leftrightarrow \quad \begin{cases} x = \dfrac{1}{\sqrt{2\omega}}(a + a^\dagger) \\ p = -\mathrm{i}\sqrt{\dfrac{\omega}{2}}(a - a^\dagger). \end{cases} \tag{2}$$

These operators satisfy the commutation relation $[a, a^\dagger] = 1$. The Hamiltonian for the harmonic oscillator is given by

$$H = \tfrac{1}{2}p^2 + \tfrac{1}{2}\omega^2 x^2 = \omega\big(a^\dagger a + \tfrac{1}{2}\big) = \omega\big(N + \tfrac{1}{2}\big), \tag{3}$$

where $N = a^\dagger a$ is the particle number operator. Units are chosen such that $\hbar = 1$ for the sake of simplicity.

The Schrödinger equation expressed in a differential form is $\mathrm{d}|\psi_t\rangle = -\mathrm{i}H|\psi_t\rangle\,\mathrm{d}t$. We extend this in the following way

$$\mathrm{d}|\psi_t\rangle = \Big\{\big[-\mathrm{i}H - \tfrac{1}{2}\lambda^2(a^\dagger - \bar{a}_t)a + \tfrac{1}{2}\lambda^2(a - \bar{a}_t)\bar{a}_t\big]\,\mathrm{d}t + \lambda(a - \bar{a}_t)\,\mathrm{d}B_t\Big\}\,|\psi_t\rangle, \tag{4}$$

where

$$\bar{a}_t = \tfrac{1}{2}\langle\psi_t|(a + a^\dagger)|\psi_t\rangle, \tag{5}$$

and λ is a constant parameter of dimension [time]$^{-1/2}$. Denoting unconditional expectation with respect to the physical probability measure $\mathbb{P}$ by $\mathbb{E}^{\mathbb{P}}[\cdot]$, the differential $\mathrm{d}B_t$ is an increment of real $\mathbb{P}$-Brownian motion with the properties that $\mathbb{E}^{\mathbb{P}}[\mathrm{d}B_t] = 0$, $(\mathrm{d}B_t)^2 = \mathrm{d}t$, and increments at different times are independent. Equation (4) can be derived (see [6]) by first assuming a state evolution equation of the form $\mathrm{d}|\phi_t\rangle = (C\,\mathrm{d}t + \lambda a\,\mathrm{d}X_t)\,|\phi_t\rangle$, where $|\psi_t\rangle = |\phi_t\rangle\langle\phi_t|\phi_t\rangle^{-1/2}$ and $\{X_t\}$ is a $\mathbb{Q}$-Brownian motion. The physical measure $\mathbb{P}$ is related to $\mathbb{Q}$ through $\mathbb{P}(A) = \mathbb{E}^{\mathbb{P}}[\mathbf{1}_A] = \mathbb{E}^{\mathbb{Q}}[\langle\phi_t|\phi_t\rangle\mathbf{1}_A]$ for some event A measurable at time t, where $\mathbf{1}_A = 1$ if A is true and 0 otherwise. This choice of physical probability measure is the counterpart to the postulate of standard quantum mechanics on the outcomes of measurement processes [6].

Note that since the state evolves according to equation (4) by the action of only the number operator and the annihilation operator, a final state with higher energy than any of those states contributing to the initial superposition $|\psi_0\rangle$ cannot occur. This ensures that as long as the initial state has finite energy, subsequent evolved states must also have finite energy.

We proceed by demonstrating that equation (4) preserves the norm of a state. Denoting $|\mathrm{d}\psi_t\rangle = \mathrm{d}|\psi_t\rangle$ we have

$$\begin{aligned}
\mathrm{d}(\langle\psi_t|\psi_t\rangle) &= \langle\,\mathrm{d}\psi_t|\psi_t\rangle + \langle\psi_t|\mathrm{d}\psi_t\rangle + \langle\,\mathrm{d}\psi_t|\mathrm{d}\psi_t\rangle \\
&= \langle\psi_t|\left[\mathrm{i}H - \tfrac{1}{2}\lambda^2 a^\dagger(a-\bar{a}_t) + \tfrac{1}{2}\lambda^2(a^\dagger-\bar{a}_t)\bar{a}_t\right]|\psi_t\rangle\,\mathrm{d}t + \langle\psi_t|\lambda(a^\dagger-\bar{a}_t)|\psi_t\rangle\,\mathrm{d}B_t \\
&\quad + \langle\psi_t|\left[-\mathrm{i}H - \tfrac{1}{2}\lambda^2(a^\dagger-\bar{a}_t)a + \tfrac{1}{2}\lambda^2(a-\bar{a}_t)\bar{a}_t\right]|\psi_t\rangle\,\mathrm{d}t + \langle\psi_t|\lambda(a-\bar{a}_t)|\psi_t\rangle\,\mathrm{d}B_t \\
&\quad + \langle\psi_t|\lambda^2(a^\dagger-\bar{a}_t)(a-\bar{a}_t)|\psi_t\rangle\,\mathrm{d}t \\
&= 0. \qquad (6)
\end{aligned}$$

For convenience we take the norm of the initial state $|\psi_0\rangle$ to be unity. Further, we make the following definitions for the conditional expectation and conditional variance of some operator O with respect to the state $|\psi_t\rangle$ at time t:

$$O_t = \langle\psi_t|O|\psi_t\rangle \qquad \text{and} \qquad V_t^O = \langle\psi_t|(O^\dagger - O_t^*)(O-O_t)|\psi_t\rangle,$$

and the conditional covariance of two operators O and O':

$$V_t^{O,O'} = \langle\psi_t|(O^\dagger - O_t^*)(O'-O_t')|\psi_t\rangle.$$

In addition, we define the operator $\Delta O_t = O - O_t$.

Let us first consider the energy of the oscillator. It is straightforward to demonstrate that the energy process $H_t = \langle\psi_t|H|\psi_t\rangle$ satisfies the evolution equation

$$\mathrm{d}H_t = -\lambda^2\omega N_t\,\mathrm{d}t + \lambda\omega\langle\psi_t|(a^\dagger a^\dagger a + a^\dagger a a - 2a^\dagger a\bar{a}_t)|\psi_t\rangle\,\mathrm{d}B_t. \qquad (7)$$

By integrating and taking the unconditional expectation, we infer that

$$\mathbb{E}^{\mathbb{P}}[H_t] = H_0 - \lambda^2\mathbb{E}^{\mathbb{P}}\left[\int_0^t \mathrm{d}u\,\omega N_u\right] = H_0 - \lambda^2\omega\int_0^t \mathrm{d}u\,\mathbb{E}^{\mathbb{P}}[N_u]. \qquad (8)$$

The second term on the right side is negative semi-definite. Therefore, energy is lost from the harmonic oscillator on average at a rate determined by λ^2. We demand that energy loss on a macroscopic scale is negligible in order to conform with the energy conservation principle. Taking the typical particle number in the state $|\psi_t\rangle$ to be of order N_0, we therefore require that $\lambda^2\omega N_0\Delta t \ll H_0$ for typical timescales Δt. Equivalently, we may say that λ must be very small in standard macroscopic units of time. In this limit we have that $\mathbb{E}^{\mathbb{P}}[H_t] \simeq H_0$, or that the expected energy is approximately conserved. In addition, having very small λ means that for a small number of particles, equation (4) can be accurately approximated by Schrödinger's equation.

2.1. State reduction

In order to see how the state reduction mechanism works we consider the stochastic processes a_t and V_t^a for the conditional expectation of the annihilation operator and the associated conditional variance:

$$\mathrm{d}a_t = -\mathrm{i}\omega a_t\,\mathrm{d}t - \tfrac{1}{2}\lambda^2 a_t\,\mathrm{d}t + \lambda\langle\psi_t|[(a+a^\dagger)a - 2\bar{a}_t a]|\psi_t\rangle\,\mathrm{d}B_t, \tag{9}$$

$$\begin{aligned}\mathrm{d}V_t^a = &-\lambda^2\{\langle\psi_t||\Delta a_t|^2|\psi_t\rangle + |\langle\psi_t|[(a+a^\dagger)a - 2\bar{a}_t a]|\psi_t\rangle|^2\}\,\mathrm{d}t\\ &+\lambda\langle\psi_t|[(a^\dagger - \bar{a}_t)|\Delta a_t|^2 + |\Delta a_t|^2(a-\bar{a}_t)]|\psi_t\rangle\,\mathrm{d}B_t.\end{aligned} \tag{10}$$

Integrating and taking the unconditional expectation of equation (10) we have

$$\begin{aligned}\mathbb{E}^{\mathbb{P}}\big[V_t^a\big] &= V_0^a - \lambda^2\mathbb{E}^{\mathbb{P}}\left[\int_0^t \mathrm{d}u\; V_u^a\right] - \lambda^2\mathbb{E}^{\mathbb{P}}\left[\int_0^t \mathrm{d}u\,\big|V_u^{(a+a^\dagger),a}\big|^2\right]\\ &= V_0^a - \lambda^2\int_0^t \mathrm{d}u\,\mathbb{E}^{\mathbb{P}}\big[V_u^a\big] - \lambda^2\int_0^t \mathrm{d}u\,\mathbb{E}^{\mathbb{P}}\big[\big|V_u^{(a+a^\dagger),a}\big|^2\big].\end{aligned} \tag{11}$$

Since the last two terms on the right side are positive semi-definite, the unconditional expectation of the variance of a cannot increase (i.e. V_t^a is a super-martingale). If we suppose that these terms are nonzero then $\mathbb{E}^{\mathbb{P}}\big[V_t^a\big] \to 0$ for large times and therefore $V_t^a \to 0$, i.e. the state approaches an a-eigenstate. Otherwise, if for some time t we have $\mathbb{E}^{\mathbb{P}}\big[V_t^a\big] = 0$ and $\mathbb{E}^{\mathbb{P}}\big[\big|V_t^{(a+a^\dagger),a}\big|^2\big] = 0$, then $|\psi_t\rangle$ at that time must be an a-eigenstate. Note that the second of these two conditions is also satisfied when $|\psi_t\rangle$ is a position eigenstate at time t. Since these are composed of an infinite number of infinitesimal energy mode contributions, we exclude this possibility.

In order to estimate the characteristic timescale for state reduction, we approximate equation (11) by freezing the stochastic terms on the right side at $t = 0$. In this approximation we find

$$\frac{\mathbb{E}^{\mathbb{P}}\big[V_t^a\big] - V_0^a}{V_0^a} \simeq -\lambda^2\left(1 + \frac{\big|V_0^{(a+a^\dagger),a}\big|^2}{V_0^a}\right)t. \tag{12}$$

Taking $V_0^a \sim V_0^{(a+a^\dagger),a} \sim \mathcal{O}(N_0)$ (corresponding, for example, to a superposition between a large excited state and the vacuum state), the reduction timescale for the variance-decreasing process can be estimated as

$$\tau_R \sim \frac{V_0^a}{\lambda^2\big|V_0^{(a+a^\dagger),a}\big|^2} \sim \frac{1}{\lambda^2 N_0}. \tag{13}$$

This must be small in standard units for macroscopic objects such that macroscopic superpositions are suppressed. For example, for an oscillator with frequency of order $10^{14}\,\mathrm{s}^{-1}$ (corresponding to visible light), if we take $N_0 = 10^{23}$ and $\hbar = 10^{-34}\,\mathrm{Js}$, then choosing $\lambda = 10^{-8}\,\mathrm{s}^{-1/2}$ would lead to energy loss at a rate of $10^{-13}\,\mathrm{Js}^{-1}$ and state reduction on a timescale of order $10^{-7}\,\mathrm{s}$. For one particle ($N_0 = 1$), energy loss is of order $10^{-36}\,\mathrm{Js}^{-1}$ and the reduction timescale is $10^{16}\,\mathrm{s}$ (10^9 yrs).

Once the system enters an a-eigenstate, equation (9) reduces to

$$\mathrm{d}a_t = \left(-\mathrm{i}\omega - \tfrac{1}{2}\lambda^2\right)a_t\,\mathrm{d}t, \tag{14}$$

with solution $a_t = a_0\exp\left\{-\mathrm{i}\omega t - \tfrac{1}{2}\lambda^2 t\right\}$. The solution decays on timescale λ^{-2} which as stated earlier must be very large in standard macroscopic units of time.

So far we have demonstrated that our state evolution equation (4) describes state reduction to a coherent state on timescale τ_R given in equation (13). We have also shown that coherent

states themselves will decay to the vacuum state on a very long timescale λ^{-2}. We conclude this subsection by confirming that stochastic probabilities match with quantum probabilities for the outcome of a simplified measurement. Let us consider the projection operator of a particle number eigenstate $P_n = |n\rangle\langle n|$. The conditional expectation of the projection operator $P_{n,t} = \langle\psi_t|P_n|\psi_t\rangle$ obeys the evolution equation

$$\mathrm{d}P_{n,t} = \lambda^2[(n+1)P_{n+1,t} - nP_{n,t}]\,\mathrm{d}t + \lambda\langle\psi_t|(a^\dagger P_n + P_n a - 2\bar{a}_t P_n)|\psi_t\rangle\,\mathrm{d}B_t, \tag{15}$$

where the terms in square brackets on the right side correspond to the background decay mechanism occurring on timescale λ^{-2}. These terms together are small when a given wavepacket is sufficiently smoothly varying in n. (For example, a wavepacket centred at $n = n'$ with a standard deviation in n of $\mathcal{O}(\sqrt{n'})$, typically has $P_{n,t} \sim \mathcal{O}(1/\sqrt{n'})$ and $[P_{n+1,t} - P_{n,t}] \sim \mathcal{O}(1/n')$, resulting in $[(n+1)P_{n+1,t} - nP_{n,t}] \sim \mathcal{O}(1)$. These orders of magnitude correspond to a minimum uncertainty coherent state wavepacket.)

Consider now an initial superposition state $|\psi_0\rangle$ consisting of the vacuum state $|0\rangle$ and some excited coherent state $|\alpha_0\rangle$. Suppose further that $\langle 0|\alpha_t\rangle \simeq 0$. We may think of this situation as corresponding to a superposition of null and positive readings on some measuring device.

After some time t where $\tau_R < t \ll \lambda^{-2}$ reduction has occurred to a coherent state. This may be either the vacuum state or $|\alpha_t\rangle$. The initial quantum probability for registering the system in the vacuum state is $P_{\mathrm{vac},0} = \langle\psi_0|P_{\mathrm{vac}}|\psi_0\rangle$ where $P_{\mathrm{vac}} = |0\rangle\langle 0|$. From equation (15) we have (upon ignoring the terms in square brackets)

$$\mathrm{d}P_{\mathrm{vac},t} \simeq \lambda\langle\psi_t|(a^\dagger P_{\mathrm{vac}} + P_{\mathrm{vac}} a - 2\bar{a}_t P_{\mathrm{vac}})|\psi_t\rangle\,\mathrm{d}B_t. \tag{16}$$

Now taking the unconditional expectation we have

$$P_{\mathrm{vac},0} \simeq \mathbb{E}^{\mathbb{P}}[P_{\mathrm{vac},t}] \simeq \mathbb{E}^{\mathbb{P}}[\mathbf{1}_{|\psi_t\rangle=|0\rangle}]. \tag{17}$$

The final approximation results from the fact that the state at time t is either the vacuum state or the approximately orthogonal excited coherent state $|\alpha_t\rangle$. This relation tells us that the initial standard quantum estimate for the probability of finding the system in the vacuum state is equal to the stochastic probability of that outcome occurring in this model. The quantum and stochastic probabilities for the other outcome must also be equal.

2.2. Numerical simulations

In order to confirm the reductive properties, we ran a numerical simulation of the quantum state evolution. We considered an initial state corresponding to an equal superposition of two a-eigenstates with eigenvalues 0 and 8 respectively. We have set the parameters to $\lambda = 0.5$ and $\omega = 1$. This choice means we observe state reduction for small numbers of particles with only a small degree of energy loss. Since $N_0 \sim 32$ we estimate the reduction timescale by equation (13) to be $\tau_R \sim 0.125$. The decay timescale is given by $\lambda^{-2} \sim 4$. These order-of-magnitude estimates are confirmed by figures 1 and 2 which show sample paths for the conditional expectation of energy and for the conditional variance in a respectively. We see that the state evolves into either one of the two possible coherent states. One of these states is the vacuum state, the other corresponds to the (slowly decaying) non-vacuum coherent state.

In addition we have estimated the probabilities of the two possible outcomes by running 100 sample paths. We find probabilities of 0.47 for the vacuum state and 0.53 for the non-vacuum state (the standard deviation of this estimate is 0.1).

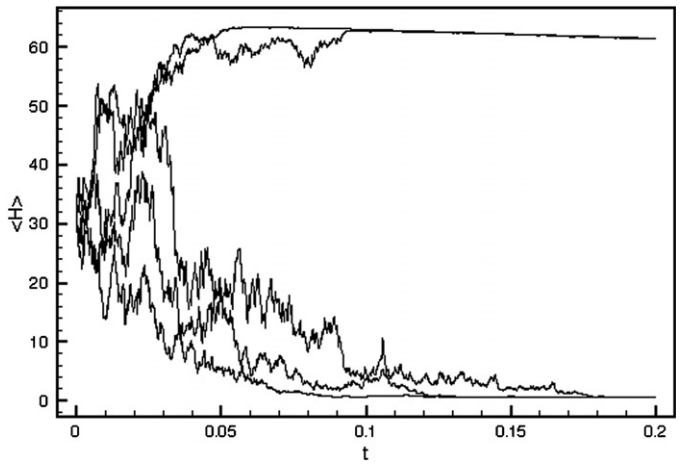

Figure 1. Conditional expectation of energy. The plot shows five realized paths for an initial state corresponding to an equal superposition of two coherent states with expected energies 0.5 and 64.5 respectively. In the cases where the state reduces to the excited coherent state we note a slow decay in energy. This is expected to occur on a timescale of order $\lambda^{-2} \sim 4$ in this example ($\lambda = 0.5$ and $\omega = 1$).

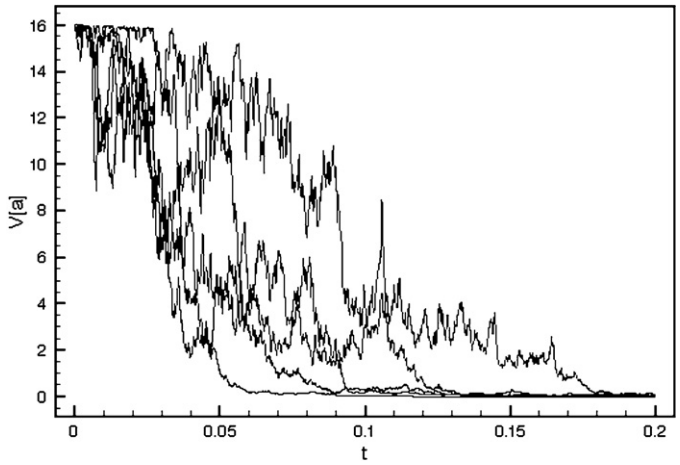

Figure 2. Conditional variance of the annihilation operator. The sample paths correspond to those in figure 1 ($\lambda = 0.5$ and $\omega = 1$).

3. Relativistic quantum field theory

Here we generalize the analysis of the previous section to the case of relativistic quantum field theory. (For a discussion of the conceptual issues surrounding the formulation of relativistic state reduction models, see [14, 24–26].) Given that experimental evidence conforms to the principle of relativistic invariance, it is natural to require this condition of our model. This has been a longstanding problem in the field of dynamical state reduction models. The reason is that while state reduction can be modelled easily enough, by coupling a stochastic process to a quantum field we generate energy at an infinite rate. We will resolve this issue by coupling

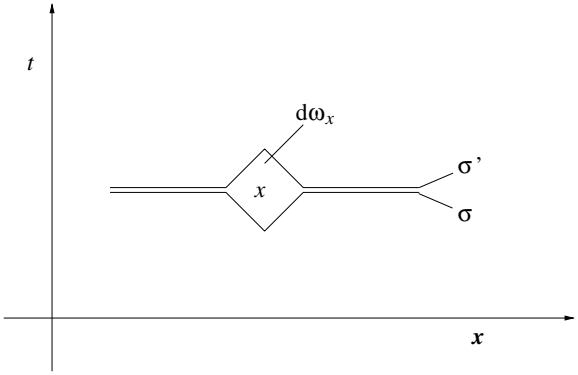

Figure 3. Evolution between space-like hyper-surfaces σ and σ'.

only the annihilation operators of the quantum field to the stochastic process (as in the case of the harmonic oscillator discussed in the previous section). This will ensure that energy cannot be created from the vacuum.

A natural formulation of relativistic quantum field theory for the consideration of an evolving state is the one due to Tomonaga and Schwinger [13, 22, 23]. We write the Hamiltonian density at spacetime point x in the form $H(x) = H_0(x) + H_{\rm int}(x)$, where H_0 is the free-field Hamiltonian and $H_{\rm int}$ is an interaction term. Then evolution of the quantum state is described by the Tomonaga equation:

$$\mathrm{i}\frac{\delta}{\delta\sigma(x)}|\Psi(\sigma)\rangle = H_{\rm int}(x)|\Psi(\sigma)\rangle. \tag{18}$$

The state is defined on some space-like three-surface σ, and functional differentiation is defined with respect to some point x lying on σ. Given two space-like surfaces σ and σ' differing only by some infinitesimal spacetime volume $\mathrm{d}\omega_x$ at point x (see figure 3), the functional derivative can be expressed as

$$\frac{\delta|\Psi(\sigma)\rangle}{\delta\sigma(x)} = \lim_{\sigma'\to\sigma}\frac{|\Psi(\sigma')\rangle - |\Psi(\sigma)\rangle}{\mathrm{d}\omega_x}. \tag{19}$$

Equation (18) describes the evolution of the quantum state in terms of incremental time-like advancements of individual points on a space-like surface. The operator $H_{\rm int}$ must be a scalar quantity in order that equation (18) has a relativistically invariant form. In addition, the constraint $[H_{\rm int}(x), H_{\rm int}(x')] = 0$ for space-like separated x and x' is imposed so that the ordering of points undergoing time-like advancement is irrelevant. We will consider the possibility of a definite ordering of all spacetime points, allowing us to break the commutation constraint.

In differential form the Tomonaga equation can be represented as follows

$$\mathrm{d}_x|\Psi(\sigma)\rangle = -\mathrm{i}H_{\rm int}(x)|\Psi(\sigma)\rangle\,\mathrm{d}\omega_x. \tag{20}$$

We proceed by generalizing this equation to a diffusion equation.

3.1. Field state diffusion equation

Previous approaches to modifying Schrödinger field dynamics have generally involved the inclusion of a white-noise field term in the Tomonaga equation (see e.g. [6]). Here we opt

to formulate our model in terms of a Gaussian process. We begin by defining $\mathrm{d}W_x$ to be an increment of some real $\mathbb{Q}$-Brownian motion with mean zero and covariance given by

$$\mathbb{E}^{\mathbb{Q}}[\mathrm{d}W_x\,\mathrm{d}W_{x'}] = \delta_{x,x'}\,\mathrm{d}\omega_x. \tag{21}$$

We may think of the Gaussian random variable $W(\sigma)$ defined on some surface σ and of $\mathrm{d}W_x$ as the incremental difference in W between two surfaces differing by some infinitesimal spacetime volume at point x.

We extend the differential Tomonaga equation to include a stochastic term as follows:

$$\mathrm{d}_x|\Phi(\sigma)\rangle = \left(-\tfrac{1}{2}\lambda^2 A^\dagger(x)A(x)\,\mathrm{d}\omega_x + \lambda A(x)\,\mathrm{d}W_x\right)|\Phi(\sigma)\rangle. \tag{22}$$

Here $A(x)$ is a scalar operator to be specified.

When using the Tomonaga picture, in order to set the initial conditions we must specify an initial state on a definite initial space-like surface σ_i. If we then wish to calculate the expected state at a later localized region in spacetime, we must specify a final space-like surface σ_f which includes this region. To describe evolution from the initial state on σ_i to the final state on σ_f we could choose any causally ordered set of intermediate space-like surfaces (we write $\sigma' > \sigma$ if σ' is nowhere in the past of any point on σ). Each surface will differ by only an incremental spacetime volume $\mathrm{d}\omega_x$ from its neighboring surfaces in the ordering. If evolution of the state from the initial to the final surface is independent of the ordering of intermediate surfaces we can say that it is independent of any specific local frame. This is true of equation (22) provided that $[A(x), A(x')] = [A(x), A^\dagger(x')] = 0$ for space-like separated x and x'.

Assuming the usual rules of Itô calculus we find

$$\mathrm{d}_x(\langle\Phi(\sigma)|\Phi(\sigma)\rangle) = 2\lambda\langle\bar{A}(x)\rangle_\sigma\langle\Phi(\sigma)|\Phi(\sigma)\rangle\,\mathrm{d}W_x, \tag{23}$$

where $\bar{A}(x) = \frac{1}{2}(A(x)+A^\dagger(x))$, $\langle\cdot\rangle_\sigma = \langle\Psi(\sigma)|\cdot|\Psi(\sigma)\rangle$ and $|\Psi(\sigma)\rangle = |\Phi(\sigma)\rangle\langle\Phi(\sigma)|\Phi(\sigma)\rangle^{-\frac{1}{2}}$ is the normalized state. The solution to this equation can be formally written as

$$\langle\Phi(\sigma_\mathrm{f})|\Phi(\sigma_\mathrm{f})\rangle = \langle\Phi(\sigma_\mathrm{i})|\Phi(\sigma_\mathrm{i})\rangle + 2\lambda\int_{\sigma_\mathrm{i}}^{\sigma_\mathrm{f}}\langle\bar{A}(x)\rangle_\sigma\langle\Phi(\sigma)|\Phi(\sigma)\rangle\,\mathrm{d}W_x \tag{24}$$

$$= \langle\Phi(\sigma_\mathrm{i})|\Phi(\sigma_\mathrm{i})\rangle\exp\left\{2\lambda\int_{\sigma_\mathrm{i}}^{\sigma_\mathrm{f}}\langle\bar{A}(x)\rangle_\sigma\,\mathrm{d}W_x - 2\lambda^2\int_{\sigma_\mathrm{i}}^{\sigma_\mathrm{f}}\langle\bar{A}(x)\rangle_\sigma^2\,\mathrm{d}\omega_x\right\}. \tag{25}$$

We next introduce the physical measure $\mathbb{P}$ such that for a random variable X, measurable on surface σ_f, the $\mathbb{P}$-expectation is given by

$$\mathbb{E}^{\mathbb{P}}[X] = \mathbb{E}^{\mathbb{Q}}\left[\frac{\langle\Phi(\sigma_\mathrm{f})|\Phi(\sigma_\mathrm{f})\rangle}{\langle\Phi(\sigma_\mathrm{i})|\Phi(\sigma_\mathrm{i})\rangle}X\right]. \tag{26}$$

The physical measure $\mathbb{P}$ assigns physical probabilities to possible measurable outcomes. We have from equation (24) that $\mathbb{P}(\Omega) = \mathbb{E}^{\mathbb{P}}[1] = 1$ as required of a probability measure. Also, as a consistency check, given the tower law of $\mathbb{Q}$-expectation, it can be shown that the tower law of $\mathbb{P}$-expectation also holds:

$$\mathbb{E}^{\mathbb{P}}[X] = \mathbb{E}^{\mathbb{P}}[\mathbb{E}^{\mathbb{P}}[X|\sigma]]. \tag{27}$$

Here σ is some surface such that $\sigma_\mathrm{f} > \sigma > \sigma_\mathrm{i}$, and by conditioning on σ we mean that all $\mathrm{d}W_x$ to the past of σ are known. It therefore makes no difference for the final outcome if we condition on some intermediate surface before taking the expectation at σ_i. The application of equation (26) therefore provides a consistent way of assigning physical probabilities to outcomes. This allows us to describe state evolution in terms of the $\mathbb{Q}$-Brownian motion before using the $\mathbb{P}$-measure to determine physical probabilities at the end of the calculation.

We can also express the state evolution equation directly in terms of a $\mathbb{P}$-Brownian motion as follows. First we choose a definite sequence of space-like hyper-surfaces $\{\sigma\}$ (with

$\sigma_f > \sigma > \sigma_i$) to support our state evolution. We then define the process $B(\sigma)$ by the solution to the following stochastic equation

$$dB_x = dW_x - 2\lambda\langle\bar{A}(x)\rangle_\sigma \, d\omega_x. \tag{28}$$

Here σ is different from its succeeding surface only by some incremental spacetime volume about x. It can be shown that $\mathbb{E}^{\mathbb{P}}[dB_x] = 0$ and $\mathbb{E}^{\mathbb{P}}[dB_x \, dB_{x'}] = \delta_{x,x'} \, d\omega_x$. Therefore dB_x is an increment of $\mathbb{P}$-Brownian motion. Finally, writing equation (22) in terms of the normalized state $|\Psi(\sigma)\rangle$ and the $\mathbb{P}$-Brownian motion dB_x we find

$$d_x|\Psi(\sigma)\rangle = (\alpha(x,\sigma)\, d\omega_x + \beta(x,\sigma)\, dB_x)|\Psi(\sigma)\rangle, \tag{29}$$

where (cf equation (4))

$$\alpha(x,\sigma) = -\tfrac{1}{2}\lambda^2(A^\dagger(x) - \langle\bar{A}(x)\rangle_\sigma)A(x) + \tfrac{1}{2}\lambda^2(A(x) - \langle\bar{A}(x)\rangle_\sigma)\langle\bar{A}(x)\rangle_\sigma, \tag{30}$$

$$\beta(x,\sigma) = \lambda(A(x) - \langle\bar{A}(x)\rangle_\sigma). \tag{31}$$

In the case where $[A(x), A(x')] = [A(x), A^\dagger(x')] = 0$ for space-like separated x and x', this evolution equation must be independent of the choice $\{\sigma\}$ by construction. On the other hand, if we allow for $[A(x), A(x')] = [A(x), A^\dagger(x')] \neq 0$, the state evolution described by equation (22) is $\{\sigma\}$-dependent and our choice of sequence $\{\sigma\}$ must be specific if the model is to give unambiguous results. Equation (29) retains its relativistically invariant form so that all observers will agree on outcomes.

We will be forced to choose operators $A(x)$ that do not commute at space-like separation and therefore we must specify a fixed sequence of evolving space-like hyper-surfaces. This might seem a significant compromise; however, it is not clear that a freedom to choose any space-like surface is desirable in a model of quantum state reduction. Consider an entangled EPR pair where one particle is measured at a space-like separation from a region where we wish to consider the state of the other particle. The state of the unmeasured particle depends on whether the surface on which it is defined has the measurement event in its past or future. Since we are free to choose this surface, the state of the unmeasured particle is ambiguous. As is argued in [14] this problem only persists for the state reduction timescale so it can be ignored for macroscopic objects. However, it is a difficulty if we intend for our state to represent the microscopic world unambiguously.

If nature were to choose the specific sequence $\{\sigma\}$ this problem could be avoided. We would have no freedom to choose the surface upon which the final state is defined. We do not suggest a rule for the choice. We only suggest that relativistic invariance could be recovered in expectation by assuming that future space-like surfaces are chosen at random from a uniform distribution over the space of all future space-like surfaces. Alternatively, we might simply be content to allow our model to break relativistic invariance in its description of state reduction. For example, the evolving surfaces could correspond to the constant time surfaces in the co-moving frame of the Universe or to a local frame defined by the matter content of the state.

Without a rule for choosing the sequence of surfaces, we must quantify the effect of making different choices. In fact, we can demonstrate that the imposed ordering of spacetime points has a negligible effect in a perturbative calculation scheme involving the coupling parameter λ. Given some operator O such that $d_x O = 0$ in the Tomonaga picture, we can use (29) to determine the dynamical equation satisfied by its conditional expectation as

$$d_x\langle O\rangle_\sigma = \langle\alpha^\dagger(\sigma)O + O\alpha(\sigma) + \beta^\dagger(\sigma)O\beta(\sigma)\rangle_\sigma \, d\omega_x + \langle\beta^\dagger(\sigma)O + O\beta(\sigma)\rangle_\sigma \, dB_x, \tag{32}$$

(where the x dependence of α and β is assumed). Integrating and taking the unconditional expectation, we find

$$\mathbb{E}^{\mathbb{P}}[\langle O\rangle_{\sigma_f}] = \langle O\rangle_{\sigma_i} + \mathbb{E}^{\mathbb{P}}\left[\int_{\sigma_i}^{\sigma_f} \langle\alpha^\dagger(\sigma)O + O\alpha(\sigma) + \beta^\dagger(\sigma)O\beta(\sigma)\rangle_\sigma \, d\omega_x\right]. \tag{33}$$

Since $\alpha \sim \mathcal{O}(\lambda^2)$ and $\beta \sim \mathcal{O}(\lambda)$, we can expand $\mathbb{E}^{\mathbb{P}}\left[\langle O\rangle_{\sigma_\mathrm{f}}\right]$ perturbatively in λ to second order by freezing the stochastic state at the initial surface σ_i, that is,

$$\mathbb{E}^{\mathbb{P}}[\langle O\rangle_{\sigma_\mathrm{f}}] \simeq \langle O\rangle_{\sigma_\mathrm{i}} + \mathbb{E}^{\mathbb{P}}\left[\int_{\sigma_\mathrm{i}}^{\sigma_\mathrm{f}} \langle \alpha^\dagger(\sigma_\mathrm{i})O + O\alpha(\sigma_\mathrm{i}) + \beta^\dagger(\sigma_\mathrm{i})O\beta(\sigma_\mathrm{i})\rangle_{\sigma_\mathrm{i}}\, \mathrm{d}\omega_x\right]. \tag{34}$$

In this approximation, even when the $A(x)$-operators do not commute at space-like separation, the result only depends on the choice of initial and final surfaces, and not on any ordering of spacetime points within the integrated region. The choice of intermediate surfaces will have no effect. We will use equivalent frozen state approximations in subsequent sections. The results will be Lorentz invariant in the sense outlined here.

We end this subsection by commenting on the 'Free Will Theorem' [27] which claims to show that relativistic dynamical reduction models are incompatible with the experimenter's free will to decide which observable to measure. In subsequent responses [28, 29], it has been argued that the resolution of this conflict can be found in nonlocality (see also [30]). Certainly equation (29) is explicitly nonlocal through its dependence on the quantum state over the entire space-like surface σ. However, as pointed out by 't Hooft [31], for models of this type we should reconsider our notion of 'free will'. For example, given some definite quantum state defined on some initial surface σ_i, and given some realized $B(\sigma)$ for every space-like surface σ to the future of σ_i, then the future quantum state is determined. This future quantum state should describe all matter including the experimenter's behaviour. If we require free will in this framework, it can only result from an inability to determine the precise initial state [31].

3.2. Scalar field theory

Having established the covariant form of the theory, we now focus on a particular frame with space-like surfaces chosen to be the constant time surfaces. We have

$$|\mathrm{d}\Psi(t)\rangle = \int_{\mathbf{x}} \mathrm{d}_x|\Psi(t)\rangle = \int_{\mathbf{x}} \mathrm{d}\mathbf{x}(\alpha(x)\,\mathrm{d}t + \beta(x)\,\mathrm{d}B_t(\mathbf{x}))|\Psi(t)\rangle, \tag{35}$$

with $\mathbb{E}^{\mathbb{P}}[\mathrm{d}B_t(\mathbf{x})\,\mathrm{d}B_{t'}(\mathbf{x}')] = \delta^3(\mathbf{x}-\mathbf{x}')\delta_{t,t'}\,\mathrm{d}t$. We use the integration subscript to avoid confusion over which variables are integrated over. In this frame, time-independent operators in the Schrödinger picture are related to time-dependent operators in the Tomonaga picture by the unitary transformation $O(t) = \exp\{\mathrm{i}H_0t\}O\exp\{-\mathrm{i}H_0t\}$, where H_0 is the free-field Hamiltonian.

We consider a real scalar field φ defined in the Tomonaga picture by

$$\varphi(x) = \int \frac{\mathrm{d}\mathbf{p}}{\sqrt{2\omega_\mathbf{p}}}\{\exp(\mathrm{i}\mathbf{p}\cdot\mathbf{x} - \mathrm{i}\omega_\mathbf{p}t)a(\mathbf{p}) + \exp(-\mathrm{i}\mathbf{p}\cdot\mathbf{x} + \mathrm{i}\omega_\mathbf{p}t)a^\dagger(\mathbf{p})\}, \tag{36}$$

with free Hamiltonian

$$\begin{aligned} H_0 &= \int \mathrm{d}\mathbf{x}\left\{\tfrac{1}{2}(\partial_t\varphi(x))^2 + \tfrac{1}{2}\nabla\varphi(x)\cdot\nabla\varphi(x) + \tfrac{1}{2}m^2\varphi^2(x)\right\} \\ &= \int \mathrm{d}\mathbf{p}\omega_\mathbf{p}\left\{a^\dagger(\mathbf{p})a(\mathbf{p}) + \tfrac{1}{2}\delta^3(\mathbf{0})\right\}, \end{aligned} \tag{37}$$

where $\omega_\mathbf{p} = \sqrt{\mathbf{p}^2+m^2}$, and the creation and annihilation operators satisfy the canonical commutation relations $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] = \delta^3(\mathbf{p}-\mathbf{p}')$ and $[a(\mathbf{p}), a(\mathbf{p}')] = 0$, respectively. The positive and negative frequency components of the field are given by

$$\varphi^+(x) = \int \frac{\mathrm{d}\mathbf{p}}{\sqrt{2\omega_\mathbf{p}}}\exp(\mathrm{i}\mathbf{p}\cdot\mathbf{x} - \mathrm{i}\omega_\mathbf{p}t)a(\mathbf{p}), \tag{38}$$

$$\varphi^-(x) = \int \frac{\mathrm{d}\mathbf{p}}{\sqrt{2\omega_\mathbf{p}}} \exp\left(-\mathrm{i}\mathbf{p}\cdot\mathbf{x} + \mathrm{i}\omega_\mathbf{p} t\right) a^\dagger(\mathbf{p}), \tag{39}$$

where $\varphi = \varphi^+ + \varphi^-$. We define

$$\alpha = -\tfrac{1}{2}\lambda^2\left(\varphi^- - \tfrac{1}{2}\langle\varphi\rangle_t\right)\varphi^+ + \tfrac{1}{2}\lambda^2\left(\varphi^+ - \tfrac{1}{2}\langle\varphi\rangle_t\right)\tfrac{1}{2}\langle\varphi\rangle_t \tag{40}$$

$$\beta = \lambda\left(\varphi^+ - \tfrac{1}{2}\langle\varphi\rangle_t\right). \tag{41}$$

Here $\langle\cdot\rangle_t = \langle\Psi(t)|\cdot|\Psi(t)\rangle$. The constant parameter λ in this model has dimension, $[\text{time}]^{-1}$. We ignore for now any other possible Hamiltonian interaction terms. Since φ^+ and φ^- do not commute, our choice of constant-time surfaces must be considered special. Although, as we have seen in the previous section, by using the frozen state approximation, our results will be independent of any specific local frame.[1]

In the same manner as (6) we can demonstrate that

$$\mathrm{d}\langle\Psi(t)|\Psi(t)\rangle = 0, \tag{42}$$

so without loss of generality we may set $\langle\Psi(t)|\Psi(t)\rangle = 1$ with the state remaining normalized for all time.

Given some generic operator $O(t)$ in the Tomonaga picture, we may ask how its conditional expectation evolves. We find (cf [15])

$$\mathrm{d}\langle O\rangle_t = \langle\,\mathrm{d}O\rangle_t + \int_\mathbf{x} \mathrm{d}\mathbf{x}\langle\alpha^\dagger O + O\alpha + \beta^\dagger O\beta\rangle_t\,\mathrm{d}t + \int_\mathbf{x}\mathrm{d}\mathbf{x}\langle\beta^\dagger O + O\beta\rangle_t\,\mathrm{d}B_t(\mathbf{x}), \tag{43}$$

where dependences on spatial coordinates are understood. The first term on the right side results from the standard unitary evolution of the operator O described by the free Hamiltonian.

Similarly, we can write an evolution equation for the conditional variance of an operator. Recalling that $\Delta O_t = O - \langle O\rangle_t$ and that the conditional variance is given by $V_t[O] = \langle|\Delta O_t|^2\rangle_t$, we find (again cf [15])

$$\begin{aligned}\mathrm{d}V_t[O] = {}& \left\langle \mathrm{d}O^\dagger \Delta O_t + \Delta O_t^\dagger\,\mathrm{d}O\right\rangle_t + \int_\mathbf{x}\mathrm{d}\mathbf{x}\langle\alpha^\dagger|\Delta O_t|^2 + |\Delta O_t|^2\alpha + \beta^\dagger|\Delta O_t|^2\beta\rangle_t\,\mathrm{d}t \\ & - \int_\mathbf{x}\mathrm{d}\mathbf{x}\langle\beta^\dagger O^\dagger + O^\dagger\beta\rangle_t\langle\beta^\dagger O + O\beta\rangle_t\,\mathrm{d}t \\ & + \int_\mathbf{x}\mathrm{d}\mathbf{x}\langle\beta^\dagger|\Delta O_t|^2 + |\Delta O_t|^2\beta\rangle_t\,\mathrm{d}B_t(\mathbf{x}).\end{aligned} \tag{44}$$

Note that the third term on the right side of equation (44) is negative semi-definite. This term is responsible for the variance reduction which we can use to demonstrate state reduction (see next subsection).

We may apply equation (43) to the total energy of the quantum field. Ignoring the vacuum energy and interactions, this is given by

$$H = \int \mathrm{d}\mathbf{p}\,\omega_\mathbf{p} a^\dagger(\mathbf{p})a(\mathbf{p}). \tag{45}$$

We find after some calculation that

$$\mathrm{d}\langle H\rangle_t = -\tfrac{1}{2}\lambda^2\langle N\rangle_t\,\mathrm{d}t + \int_\mathbf{x}\mathrm{d}\mathbf{x}\langle\beta^\dagger H + H\beta\rangle_t\,\mathrm{d}B_t(\mathbf{x}), \tag{46}$$

[1] An alternative suggestion that we have explored is to remove the on-shell constraint from the field creation and annihilation operators. This enables us to construct local scalar field operators which do commute at space-like separation. The hope is that a state with only on-shell excitations might enforce the on-shell condition. However, we have been unable to prevent off-shell excitations from occurring (including faster-than-light modes).

where

$$N = \int \mathrm{d}\mathbf{p}\, a^{\dagger}(\mathbf{p}) a(\mathbf{p}). \tag{47}$$

Integrating and taking the unconditional expectation of the energy process at time t we have

$$\mathbb{E}^{\mathbb{P}}[\langle H\rangle_t] = \langle H\rangle_0 - \tfrac{1}{2}\lambda^2 \int_0^t \mathrm{d}u\, \mathbb{E}^{\mathbb{P}}[\langle N\rangle_u], \tag{48}$$

(cf equation (8)). Since $\langle N\rangle_t$ is nonnegative, it follows from (48) that energy is lost on average as a result of coupling the quantum field to a classical stochastic process. However, the energy loss is finite and can be made negligible by an appropriate choice of λ. This is to be contrasted with some of the previous attempts to construct a relativistic state reduction model [13–15], where the energy density is seen to increase at an infinite rate. The reason that we do not see an infinite rate of energy density creation can be traced back to the fact that the classical stochastic process is not coupled to the particle creation operator and therefore cannot drive particle creation from the vacuum.

Stochastic movements in the energy process will cease when the quantum state is an eigenstate of the operator φ^{+}. When this occurs, the final term on the right side of equation (46) goes to zero.

We can approximate equation (48) to $\mathcal{O}(\lambda^2)$ by freezing the stochastic terms on the right side at time $t = 0$. This gives

$$\mathbb{E}^{\mathbb{P}}[\langle H\rangle_t] \simeq \langle H\rangle_0 - \tfrac{1}{2}\lambda^2 \langle N\rangle_0 t \tag{49}$$

This result depends on the initial state and on the integrated region of spacetime between the initial and final space-like hyper-surfaces. However, no ordering of spacetime points is required.

3.3. Quantum field state reduction

To see the reductive properties, we consider the particle annihilation operator $a(\mathbf{p})$. Using equation (43) we find

$$\mathrm{d}\langle a(\mathbf{p})\rangle_t = -\mathrm{i}\omega_{\mathbf{p}}\langle a(\mathbf{p})\rangle_t\, \mathrm{d}t - \frac{\lambda^2}{4\omega_{\mathbf{p}}}\langle a(\mathbf{p})\rangle_t\, \mathrm{d}t + \lambda \int_{\mathbf{x}} \mathrm{d}\mathbf{x}\langle(\varphi - \langle\varphi\rangle_t)\, a(\mathbf{p})\rangle_t\, \mathrm{d}B_t(\mathbf{x}). \tag{50}$$

Similarly using equation (44) and taking the unconditional expectation we have

$$\begin{aligned}\mathbb{E}^{\mathbb{P}}[V_t[a(\mathbf{p})]] &= V_0[a(\mathbf{p})] - \frac{\lambda^2}{2\omega_{\mathbf{p}}}\mathbb{E}^{\mathbb{P}}\left[\int_0^t \mathrm{d}u\, V_u[a(\mathbf{p})]\right] \\ &\quad - \lambda^2\mathbb{E}^{\mathbb{P}}\left[\int_0^t \mathrm{d}u \int_{\mathbf{x}} \mathrm{d}\mathbf{x}|\langle(\varphi - \langle\varphi\rangle_t)\, a(\mathbf{p})\rangle_t|^2\right] \\ &= V_0[a(\mathbf{p})] - \frac{\lambda^2}{2\omega_{\mathbf{p}}}\int_0^t \mathrm{d}u\, \mathbb{E}^{\mathbb{P}}[V_u[a(\mathbf{p})]] \\ &\quad - \lambda^2 \int_0^t \mathrm{d}u\, \mathbb{E}^{\mathbb{P}}\left[\int_{\mathbf{x}} \mathrm{d}\mathbf{x}|\langle(\varphi - \langle\varphi\rangle_t)\, a(\mathbf{p})\rangle_t|^2\right].\end{aligned} \tag{51}$$

Again we find that the conditional variance for the annihilation operator is a super-martingale. The expected variance decreases with time and the quantum state evolves towards an eigenstate of the annihilation operator. If we freeze the stochastic terms on the right side of equation (51), we find

$$\mathbb{E}^{\mathbb{P}}[V_t[a(\mathbf{p})]] \simeq V_0[a(\mathbf{p})] - \frac{\lambda^2}{2\omega_{\mathbf{p}}} V_0[a(\mathbf{p})]t - \lambda^2 \int_0^t \mathrm{d}u \int_{\mathbf{x}} \mathrm{d}\mathbf{x}|\langle(\varphi - \langle\varphi\rangle_0)\, a(\mathbf{p})\rangle_0|^2. \tag{52}$$

Note that, as in equation (49), the right side is independent of the ordering of spacetime points and therefore independent of the intermediate space-like hyper-surfaces we have chosen to support our state evolution. We may estimate the timescale for collapse in the same manner as equations (12) and (13) by taking $V_0[a(\mathbf{p})] \sim N_0(\mathbf{p}) = \langle a^\dagger(\mathbf{p})a(\mathbf{p})\rangle_0$ and

$$\int_{\mathbf{x}} \mathrm{d}\mathbf{x} |\langle(\varphi - \langle\varphi\rangle_t)\, a(\mathbf{p})\rangle_t|^2 \sim \int \mathrm{d}\mathbf{p}' \frac{N_0(\mathbf{p}')N_0(\mathbf{p})}{2\omega_{\mathbf{p}'}}, \tag{53}$$

from which we find

$$\tau_R \sim \frac{1}{\lambda^2 \int \mathrm{d}\mathbf{p}' N_0(\mathbf{p}')/(2\omega_{\mathbf{p}'})}. \tag{54}$$

As in the harmonic oscillator case, it is the third term on the right side of equation (51) that leads to variance reduction for macroscopic energy scales. The reduction time is inversely proportional to the total number of excitations in all modes. This will lead to rapid reduction for large scale excitations. Each mode tends towards a coherent state. As this occurs, we expect that the field tends towards classical behaviour.

3.4. Fermionic state reduction

Here we introduce a fermionic field coupled to our proposed scalar field theory in order to consider an induced state reduction in the fermionic sector. To see how this works let us set λ to zero for now and consider an interaction Hamiltonian of the type

$$H_{\mathrm{int}}(t) = \int_{\mathbf{x}} \mathrm{d}\mathbf{x}\, j(x)\varphi(x). \tag{55}$$

Here j is some Hermitian current operator associated with the fermionic matter field. From equation (36) we have

$$\begin{aligned} H_{\mathrm{int}}(t) &= \int_{\mathbf{x}} \mathrm{d}\mathbf{x}\, j(x) \int \frac{\mathrm{d}\mathbf{p}}{\sqrt{2\omega_{\mathbf{p}}}} \{\exp(\mathrm{i}\mathbf{p}\cdot\mathbf{x} - \mathrm{i}\omega_{\mathbf{p}}t)a(\mathbf{p}) + \exp(-\mathrm{i}\mathbf{p}\cdot\mathbf{x} + \mathrm{i}\omega_{\mathbf{p}}t)a^\dagger(\mathbf{p})\} \\ &= \int \frac{\mathrm{d}\mathbf{p}}{\sqrt{2\omega_{\mathbf{p}}}} \{j^\dagger(\mathbf{p},t)\exp(-\mathrm{i}\omega_{\mathbf{p}}t)a(\mathbf{p}) + j(\mathbf{p},t)\exp(\mathrm{i}\omega_{\mathbf{p}}t)a^\dagger(\mathbf{p})\}. \end{aligned} \tag{56}$$

Furthermore, we can formally solve the Tomonaga equation to find

$$|\Psi(t)\rangle = \exp\left\{-\mathrm{i}\int_0^t \mathrm{d}u\, H_{\mathrm{int}}(u)\right\}|\Psi(0)\rangle. \tag{57}$$

Now suppose that the fermionic state undergoes some spatial transfer of charge such that a pulse of current occurs. If the fermionic state is a j-eigenstate, we have

$$|\Psi(t)\rangle = \exp\left\{\int \mathrm{d}\mathbf{p}(\alpha(\mathbf{p},t)a^\dagger(\mathbf{p}) - \alpha^*(\mathbf{p},t)a(\mathbf{p}))\right\}|\Psi(0)\rangle, \tag{58}$$

where the complex number α is given by

$$\alpha(\mathbf{p},t) = -\mathrm{i}\int_0^t \mathrm{d}u \frac{j(\mathbf{p},u)}{\sqrt{2\omega_{\mathbf{p}}}} \exp(\mathrm{i}\omega_{\mathbf{p}}u), \tag{59}$$

and $j(\mathbf{p},t)$ is the current eigenvalue at time t. Using the commutation relations for the creation and annihilation operators, and assuming that the initial φ state is unexcited, we find

$$\begin{aligned} a(\mathbf{p}')|\Psi(t)\rangle &= a(\mathbf{p}')\exp\left\{\int \mathrm{d}\mathbf{p}(\alpha(\mathbf{p},t)a^\dagger(\mathbf{p}) - \alpha^*(\mathbf{p},t)a(\mathbf{p}))\right\}|\Psi(0)\rangle \\ &= \exp\left\{\int \mathrm{d}\mathbf{p}(\alpha(\mathbf{p},t)a^\dagger(\mathbf{p}) - \alpha^*(\mathbf{p},t)a(\mathbf{p}))\right\}(a(\mathbf{p}') + \alpha(\mathbf{p}',t))|\Psi(0)\rangle \\ &= \alpha(\mathbf{p}',t)|\Psi(t)\rangle. \end{aligned} \tag{60}$$

The final state is a φ-coherent state with eigenvalue α (cf section 3.4 in [21]). This demonstrates that coherent states in φ are associated with j-eigenstates in the matter field. Reduction to a φ-coherent state should therefore induce reduction to a j-eigenstate in the fermionic sector.

It is tempting to associate φ with a gauge field such as the photon field or some proposed graviton field. The current j would then relate to a conserved charge, e.g. electric charge or energy–momentum. Such charge densities are a natural description of macroscopic observables.

4. Conclusions

The key advance of this paper has been to develop an alternative model of state reduction in relativistic quantum field theory which does not suffer from the infinite rates of energy density increase seen in some previous proposals. We have outlined a model requiring just one extra parameter in addition to those of standard quantum theories in order to simultaneously describe the quantum behaviour of individual excitations and the definite behaviour of macroscopic objects.

In our approach, by having no coupling between the classical stochastic field and the particle creation operator, we ensure that the evolution equation cannot randomly create particles from the vacuum. Our model features only a coupling between the stochastic field and the particle annihilation operator. This is appealing for two further reasons. First, it leads to a reduction to coherent states. As coherent states saturate the bound of the Heisenberg uncertainty relation, they make a natural choice as a quantum counterpart to an idealized classical state. Second, by applying this mechanism to a bosonic field coupled to a fermionic field, we can induce state reduction to some charge density basis in the fermionic sector. The model requires the specification of a preferred set of space-like hyper-surfaces supporting the time-like state evolution. This breaks relativistic invariance. However, our perturbative calculations show no deviation from relativistic invariance to second order in λ.

The ideas presented in this paper could be applied to the photon field or to a proposed graviton field in order to see state reduction to a conserved electric charge or energy–momentum basis in the associated matter fields. Since the model predicts an energy loss which could be significant in high-density highly accelerating matter environments, there may be the possibility of experimental investigation, e.g., by looking at the decay of high intensity electromagnetic waves or through the detection of gravitational waves.

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